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ON POSITION CONTROL IN DISTRIBUTED-PARAMETER SYSTEMS*

A. I. KOROTKII and Iu. S. OSIPOV

Position control problems under conditions of indeterminacy or conflict are studied for certain classes of parabolic and hyperbolic systems. The problems are treated as differential games in suitable functional spaces /1—4/. Necessary and sufficient conditions for the solvability of the problems and a method for constructing the desired controls are indicated. Essential use was made of representation of the system's motion as a Fourier series when solving similar problems in /5—7/. In solving the problems being examined below arguments not relying on such a representation are used. This permits the consideration of certain new classes of distributed-parameter systems (in particular, those nonstationary and nonlinear in the phase variable) for which an analog of the well-known alternative proves valid and for which the controls solving the problem can be constructed as extremal strategies /1—4/. The article borders on the studies in /8—12/.

1. We consider a controlled system whose state at each instant t from a prescribed interval $[t_0, \vartheta]$ is characterized by a scalar function $y(t, \cdot) = y(t_0, x)$ defined in a domain Ω of space R^n, $n \geqslant 1$, with boundary Γ. The system is subject to the control $u = u(t, x)$ and the noise $v = v(t, x)$ with constraints $u(t, \cdot) \in P(t)$ and $v(t, \cdot) \in G(t)$, where $P(t)$ and $G(t)$ are certain collections of vector-valued functions defined on Ω. The system's dynamics are described by the relations

$$y_t = \frac{\partial}{\partial x_j}\left(a_{ij}(t, x)\frac{\partial y}{\partial x_i}\right) + f(t, x, y, u, v) \text{ in } Q = (t_0, \vartheta) \times \Omega \tag{1.1}$$

$$\sigma_1 \frac{\partial y}{\partial N} + \sigma_2(t, x) y = 0 \text{ in } \Sigma = (t, \vartheta) \times \Gamma \tag{1.2}$$

$$y(t_0, x) = y_0 \text{ in } \Omega \tag{1.3}$$

Under the constraints specified on the resources of u and v we are asked to find a method for forming the control $u(v)$ by the feedback principle $u[t] = u(t, x, y[t, \cdot])$ $(v[t] = v(t, x, y[t, \cdot]))$, ensuring (excluding) the transition of system (1.1)—(1.3) into a specified state set under any admissible realizations of control $v(u)$, the prescribed phase contraints being observed during the transition.

Let us specify the problem statement. We take it that the sets Ω, Γ, $P(t)$, $G(t)$ and the functions a_{ij} satisfy the constraints indicated in /6/, the coercivity condition is fulfilled uniformly in t, and that $\partial a_{ij} / \partial t \in L_\infty(Q)$. We assume as well that function f is measurable in (t, x) on $(t_0, \vartheta) \times \Omega$ and is continuous in (y, u, v) on $R \times R^{m_1} \times R^{m_2}$, that for every choice of $u \in P(t_0, \vartheta)$ and $v \in G(t_0, \vartheta)$ the function $f(t, x, y, u, v)$ satisfies a Lipschitz condition in y for almost all (t, x), and that $f(t, x, 0, u, v) \in L_2(Q)$ and $\|f(t, x, 0, u, v)\|_{L_2(Q)} \leqslant C$ (the Lipschitz constant and C are independent of the choice of u and v). We assume further that σ_1 is either 0 or 1 and that function $\sigma_2 = 1$ when $\sigma_1 = 0$; $\partial\sigma_2 / \partial t \in L_\infty(\Sigma)$, $\sigma_2 \geqslant 0$; $y_0 \in \Phi$, where Φ is $W_{0,2}^1(\Omega)$ when $\sigma_1 = 0$ and is $W_2^1(\Omega)$ when $\sigma_1 = 1$. Here $P(t_1, t_2)$ $(G(t_1, t_2))$ is the set of all functions $t \to P(t)$ $(G(t))$ measurable on $[t_1, t_2] \subseteq [t_0, \vartheta)$. According to the theorem on the measurable selection these sets are nonempty /13/. Measurability and integrability is everywhere understood in the Lebesgue sense; derivatives are understood in the generalized sense (see /14—16/, for instance).

A rule U that associates some nonempty subset $U(t_1, t_2, y) \subseteq P(t_1, t_2)$ with every triple $\{t_1, t_2, y\}$, $t_0 \leqslant t_1 < t_2 \leqslant \vartheta$, $y \in L_2(\Omega)$, is called a strategy. Let Δ be a finite partitioning of $[t_0, \vartheta]$ by points $t_0 = \tau_0 < \ldots < \tau_m = \vartheta$, $d\Delta = \max_i(\tau_{i+1} - \tau_i)$. A motion $y[t]_\Delta = y[t; t_0, y_0, U]_\Delta$, $t_0 \leqslant t \leqslant \vartheta$, of system (1.1)—(1.3) from position $\{t_0, y_0\}$, corresponding to strategy U and partitioning Δ, is the name given to every function $y[t]_\Delta$ from $W_{2,0}^1(Q)$ when $\sigma_1 = 0$ and from $W_2^1(Q)$ when $\sigma_1 = 1$, equalling y_0 when $t = t_0$ and satisfying the identity

$$\int_Q \left(\frac{\partial y[t]_\Delta}{\partial t}\eta + a_{ij}\frac{\partial y[t]_\Delta}{\partial x_i}\frac{\partial \eta}{\partial x_j}\right) dx\,dt + \sigma_1 \int_\Sigma \sigma_2 y[t]_\Delta \eta\, d\Gamma\, dt = \int_Q f(t, x, y[t]_\Delta, u[t], v[t])\eta\, dx\, dt$$

for every function η of the same class as $y[t]_\Delta$; $u[\cdot] \in U(\tau_i, \tau_{i+1}, y[\tau_i]_\Delta)$ and $v[\cdot] \in G(\tau_i, \tau_{i+1})$ on each interval $[\tau_i, \tau_{i+1})$. It can be shown that the set of motions introduced is nonempty

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/14—16/ and that the function $y[t]_\Delta$ is continuous with respect to $t \in [t_0, \vartheta]$ in the weak topology of Φ.

By $y(t; t_0, y_0, u, v)$ we denote the program motion corresponding to the initial position $\{t_0, y_0\}$ and to the functions $u \in P(t_0, \vartheta)$ and $v \in G(t_0, \vartheta)$. The strategy V and the motions corresponding to parameter v are defined analogously. Let M and N be sets from $[t_0, \vartheta] \times \Phi$. The original control problems can now be stated as follows /6/.

Problem 1.1. Construct a strategy U with the property: for any number $\varepsilon > 0$ a number $\delta > 0$ can be found such that the contact condition

$$\rho(\{t_*, y[t_*]_\Delta\}, M) = \inf_{\{t, z\} \in M} (|t_* - t|^2 + \|y[t_*]_\Delta - z\|_{L_2}^2)^{1/2} \leqslant \varepsilon, \quad \rho(\{\tau, y[\tau]_\Delta\}, N) \leqslant \varepsilon, \ t_0 \leqslant \tau \leqslant t_*$$

is fulfilled at some instant $t_* = t(y[\cdot]_\Delta) \in [t_0, \vartheta]$ for each motion $y[t]_\Delta = y[t; t_0, y_0, U]_\Delta$ with $d\Delta \leqslant \delta$.

Problem 1.2. Construct a strategy V with the property: numbers $\varepsilon > 0$ and $\delta > 0$ exist such that the contact condition is not fulfilled for each motion $y[t]_\Delta = y[t; t_0, y_0, V]_\Delta$ with $d\Delta \leqslant \delta$.

We indicate solvability conditions for the problems and a method for constructing the strategies desired.

Condition 1.1. If the sequences $\{u_k\}$ and $\{v_k\}$ converge weakly to u and v in $L_2([t_0, \vartheta]; L_2^{m_1}(\Omega))$ and $L_2([t_0, \vartheta]; L^{m_2}{}_2(\Omega))$, respectively, then from $\{y(t; t_0, y_0, u_k, v_k)\}$ we can pick out a subsequence converging to $y(t; t_0, y_0, u, v)$ in $C([t_0, \vartheta]; L_2(\Omega))$.

Condition 1.2. The following condition is fulfilled (see /3/):

$$\min_{P(t_1, t_2)} \max_{G(t_1, t_2)} \int_{t_1}^{t_2} \langle z, f(t, x, y, u, v)\rangle_{L_2} dt = \max_{G(t_1, t_2)} \min_{P(t_1, t_2)} \int_{t_1}^{t_2} \langle z, f(t, x, y, u, v)\rangle_{L_2} dt$$

for any t_1 and t_2, $t_0 \leqslant t_1 < t_2 \leqslant \vartheta$, z and y from $L_2(\Omega)$.

Let K be some set from $[t_0, \vartheta] \times \Phi$. By the symbol U^e we denote a strategy (and we say that is is extremal to K) of the following kind. If the section $K(t_1) = \varnothing$, then $U^e(t_1, t_2, y)$ is an arbitrary subset from $P(t_1, t_2)$. If $K(t_1) \neq \varnothing$, then $U^e(t_1, t_2, y) = \{u^e\}$, where u^e is a function with the property: sequences $\{u_k\} \subset P(t_1, t_2)$ and $\{y_k\} \subset K(t_1)$ exist such that $\lim_k \|y - y_k\|_{L_2} = \inf \{\|y - z\|_{L_2} \mid z \in K(t_1)\}$, $u_k \to u^e$ weakly in $L_2([t_1, t_2]; L^{m_1}{}_2(\Omega))$, and

$$\max_{G(t_1, t_2)} \int_{t_1}^{t_2} \langle y - y_k, f(t, x, y_k, u_k, v)\rangle_{L_2} dt = \min_{P(t_1, t_2)} \max_{G(t_1, t_2)} \int_{t_1}^{t_2} \langle y - y_k, f(t, x, y_k, u, v)\rangle_{L_2} dt$$

Strategy V^e is defined analogously. By the symbol V_0 we denote a strategy V that is upper-semicontinuous with respect to variation of y in the metric of $L_2(\Omega)$. Let $W = W(M, N)$ be the set of all pairs $\{t, y\}$ from $[t_0, \vartheta] \times \Phi$, from which, as from the initial pairs, Problem 1.2 is unsolvable in the class of strategies V_0.

Theorem 1.1. When conditions 1.1 and 1.2 are fulfilled either Problem 1.1 or Problem 1.2 is solvable from every initial position $\{t_0, y_0\}$. Problem 1.1 (1.2) is solvable if and only if $\{t_0, y_0\} \in W$ ($\{t_0, y_0\} \notin W$).

2. Let us consider a controlled system whose state at each instant t of the interval $[t_0, \vartheta]$ is characterized by a scalar function $y(t, \cdot) = y(t, x)$ defined in Ω and by the rate $y_t(t, \cdot) = \partial y(t, x) / \partial t$ of variation of this function. The system is subject to a control $u = u(t)$ and a noise $v = v(t)$ with constraints $u(t) \in P$ and $v(t) \in G$, where P and G are convex compacta in R^{m_1} and R^{m_2}, respectively. The system's dynamics are described by the relations

$$y_{tt} = \frac{\partial}{\partial x_j}\left(a_{ij}(x) \frac{\partial y}{\partial x_i}\right) + f(t, x, y, y_x, y_t, \omega) \ \text{in } Q \tag{2.1}$$

$$\sigma_1 \frac{\partial y}{\partial N} + \sigma_2 y = 0 \ \text{in } \Sigma \tag{2.2}$$

$$y(t_0, x) = y_0, \ y_t(t_0, x) = y_1 \ \text{in } \Omega \tag{2.3}$$

$$w^{\cdot} = g(t, w, u, v), \ w(t_0) = w_0 \tag{2.4}$$

Under the constraints prescribed on the resources of u and v we are asked to find a method for forming the force $u(v)$ by the feedback principle

$$u[t] = u(t, y[t, \cdot], y_t[t, \cdot]), \quad (v[t] = v(t, y[t, \cdot], y_t[t, \cdot]))$$

ensuring (excluding) the transition of the system's state into a specified set, the prescribed phase constraints being observed during the transition.

Let us specify the problem statement. We take it that the following conditions are fulfilled: the a_{ij} satisfy the constraints from /6/; σ_1 and σ_2 are constants, $\sigma_1\sigma_2 = 0$; $g \in C([t_0, \vartheta] \times R^m \times P \times G)$ and locally satisfies a Lipschitz condition in w (see /3/); for every choice of measurable functions $u = u(\cdot)$ and $v = v(\cdot)$ with constraints $u(t) \in P$ and $v(t) \in G$ and for almost all $t \in [t_0, \vartheta]$ (the sets of all such functions are denoted $P(t_0, \vartheta)$ and $G(t_0, \vartheta)$ respectively) the absolutely continuous solutions $w(t)$ of system (2.4) are continuable onto $[t_0, \vartheta]$ and are uniformly bounded in $C([t_0, \vartheta]; R^m$, $y_0 \in \Phi$, $y_1 \in L_2(\Omega)$. We assume that function $f(t, x, y, p, q, w)$ is continuous in all variables on $[t_0, \vartheta] \times \Omega \times R \times R^n \times R \times R^m$, has continuous derivatives $\partial f / \partial t$, $\partial f / \partial y$, $\partial f / \partial p_k$, $\partial f / \partial q$, $\partial f / \partial w_k$, $|f(t, x, 0, 0, 0, w)| \leqslant f_D(x) \in L_2(\Omega)$ when $(t, x, w) \in [t_0, \vartheta] \times \Omega \times D$, D is a bounded set in R^m, and

$$|\partial f / \partial y, \ \partial f / \partial p_k, \ \partial f / \partial q| \leqslant L, \quad |\partial f / \partial t, \ \partial f / \partial w_k| \leqslant c_1 + c_2 |y| + c_3 \|p\|_{R^n} + c_4 |q|$$

where L and $c_1, \ldots, c_4$ are absolute constants.

For chosen y_0, y_1, w_0, u and v, by a solution of (2.1)—(2.3) we mean a function $y[t] = y[t, x; t_0, y_0, y_1, w_0, u, v]$ from $W_2^{2,1}(Q) \cap W_{2,0}^1(Q)$ when $\sigma_1 = 0$ and from $W_2^{2,1}(Q)$ when $\sigma_2 = 0$, satisfying (2.3) and the integral identity

$$\int_Q (y[t]_{tt}\eta + a_{ij} y[t]_{x_i}\eta_{x_j})\, dx\, dt = \int_Q f(t, x, y[t], y[t]_x, y[t]_t, w[t])\, \eta\, dx\, dt \tag{2.5}$$

for every function η of the same class as $y[t]$; $w[t]$ is a solution of (2.4). The set of solutions introduced is nonempty /14—16/, and

$$y[t] \in C([t_0, \vartheta]; \Phi), \quad y[t]_t \in C([t_0, \vartheta]; L_2(\Omega))$$

Let H be the space $\Phi \times L_2(\Omega) \times R^m$ with norm $\|\{y, y_t, w\}\| = (\|y\|_\Phi^2 + \|y_t\|_{L_2}^2 + \|w\|_{R^n}^2)^{1/2}$.

A rule U that associates some nonempty subset $U(t, h) \subseteq P$ with every pair $\{t, h\}$, $t_0 \leqslant t \leqslant \vartheta$, $h \in H$, is called a strategy. A function

$$z[t]_\Delta = z[t; t_0, h_0, U]_\Delta = \{y[t]_\Delta, y[t]_{\Delta t}\}, \quad t_0 \leqslant t \leqslant \vartheta$$

where $y[t]_\Delta$ satisfies (2.5), is called a motion of the system from the position $\{t_0, h_0\}$, $h_0 = \{y_0, y_1, w_0\}$, corresponding to strategy U and to partitioning Δ; and on $[\tau_i, \tau_{i+1})$

$$u[\cdot] = u \in U(\tau_i, \{z[\tau_i]_\Delta, w[\tau_i]_\Delta\}), \ v[\cdot] \in G(t_0, \vartheta)$$

Strategy V and the motion corresponding to parameter v are defined analogously. Suppose that sets M and N are specified in $[t_0, \vartheta] \times \Phi \times L_2(\Omega)$. The original problems can be formalized in the following manner.

Problem 2.1. Construct a strategy U with the property; for any $\varepsilon > 0$ a number $\delta > 0$ can be found such that the contact condition

$$\rho(\{t_*, z[t_*]_\Delta\}, M) = \inf_{\{t, z\} \in M} (|t_* - t|^2 + \|y[t_*] - y\|_\Phi^2 + \|y[t_*]_{\Delta t} - y_t\|_{L_2}^2)^{1/2} \leqslant \varepsilon, \quad \rho(\{\tau, z[\tau]_\Delta\}, N) \leqslant \varepsilon, \quad t_0 \leqslant \tau \leqslant t_*$$

is fulfilled at some instant $t_* = t(z[\cdot]_\Delta) \in [t_0, \vartheta]$ for each motion $z[t]_\Delta = z[t; t_0, h_0, U]_\Delta$ with $d\Delta \leqslant \delta$.

Problem 2.2. Construct a strategy V with the property: numbers $\varepsilon > 0$ and $\delta > 0$ can be found such that the contact condition is not fulfilled for each motion $z[t]_\Delta = z[t; t_0, h_0, V]_\Delta$ with $d\Delta \leqslant \delta$.

We state the main results.

Condition 2.1. If $P(t_0, \vartheta) \ni u_k \to u \in P(t_0, \vartheta)$ weakly in $L_2([t_0, \vartheta]; R^{m_1})$ and $G(t_0, \vartheta) \ni v_k \to v \in G(t_0, \vartheta)$ weakly in $L_2([t_0, \vartheta]; R^{m_2})$, then $\{z[t; t_0, h_0, u_k, v_k], w[t; t_0, h_0, u_k, v_k]\} \to \{z[t; t_0, h_0, u, v], w[t; t_0, h_0, u, v]\}$ in $C([t_0, \vartheta]; H)$.

Condition 2.2. The saddle point condition

$$\min_P \max_G \langle s, g(t, w, u, v)\rangle_{R^n} = \max_G \min_P \langle s, g(t, w, u, v)\rangle_{R^n}$$

is fulfilled for any $t \in [t_0, \vartheta]$ and for s and w from R^m.

Let K be some set from $[t_0, \vartheta] \times H$. By the symbol U^e we denote a strategy (and we say that it is extremal to K) of the following kind. If the section $K(t) = \phi$, then $U^e(t, h)$ is an arbitrary subset from P. If $K(t) \neq \varnothing$, then $U^e(t, h) = \{u^e\} \subseteq P$, where u^e possesses the property: sequences $\{u_k\} \subset P$ and $\{h_k\} \subset K(t)$ exist such that

$$\lim_k \|h - h_k\|_H = \inf_{q \in K(t)} \|h - q\|_H, \quad u_k \to u^e \text{ in } R^{m_1}, \quad \max_G \langle w - w_k, g(t, w, u_k, v)\rangle_{R^m} = \min_P \max_G \langle w - w_k, g(t, w, u, v\rangle_{R^m}$$

Strategy V^e is defined analogously. Let $W = W(M, N)$ be the set of all pairs $\{t, h\}$ from $[t_0, \vartheta] \times H$, from which, as from the initial pairs, Problem 2.2 is unsolvable in the class of strategies V_0, where V_0 is a strategy V upper-semicontinuous with respect to the variation of h in the metric of H.

Theorem 2.1. When Conditions 2.1 and 2.2 are fulfilled either Problem 2.1 or Problem 2.2 is solvable from every initial position $\{t_0, h_0\}$. Problem 2.1 (2.2) is solvable if and only if $\{t_0, h_0\} \in W$ ($\{t_0, h_0\} \notin W$).

3. Games on the minimax-maximin of a functional $\varphi(y[\vartheta])$ (or of functionals reducing to such; see Chapter 4 in /3/) continuous in the system's phase space, as an example, can be reduced to the differential games we have considered. In this connection an alternative

statement analogous to /3/ holds, and the optimal minimax and maximin strategies can be constructed as strategies extremal to certain stable bridges /3/.

In conclusion we indicate the possibility of approximating the problems considered by certain finite-dimensional systems. An approximation based on the Galerkin method, on the method of lines (i.e., the discretization is implemented only along the one variable x), or on the difference method with a scheme analogous to that in /11/ is possible for the problems from Sect.1. An approximation based on the Galerkin method or on the method of lines is possible for the problems from Sect.2. Approximation theorems analogous to those in /11/ hold; their sense is as follows: with any preassigned accuracy $\varepsilon > 0$ we can find the appropriate approximation and a control method for the original system (projected from the strategy solving the corresponding problem for the given approximation), which lead to the solving of the original problem with accuracy ε.

Notes. 1°. Under the constraints indicated on the parameters of system (1.1)— (1.3) the bundle of trajectories $\{y(t; t_0, y_0, u, v)\}$ starting from position $\{t_0, y_0\}$ and corresponding to all possible controls $u \in P(t_0, \vartheta)$ and $v \in G(t_0, \vartheta)$ is precompact in $C([t_0, \vartheta]; L_2(\Omega))$. Therefore, for the fulfillment of Condition 1.1 it is sufficient that function f possess the property: for every fixed function $y \in C([t_0, \vartheta]; L_2(\Omega))$ the weak convergence of sequences $\{u_k\}$ and $\{v_k\}$ to u and v, respectively, implies the convergence weak in $L_2(Q)$ of sequence $\{f(t, x, y, u_k, v_k)\}$ to $f(t, x, y, u, v)$. The bundle of trajectories of system (2.1)— (2.4) is precompact in $C([t_0, \vartheta]; H)$. For the fulfillment of Condition 2.1 it is sufficient that the weak convergence of $\{u_k\}$ and $\{v_k\}$ to u and v, respectively, imply the convergence of solutions $\{w[t; t_0, w_0, u_k, v_k]\}$ to $w[t; t_0, w_0, u, v]$ in $C([t_0, \vartheta]; R^m)$ and that boundary $\Gamma \in C^2$ when $\sigma_2 = 0$. See /3/ on the matter of the fulfillment of conditions of type 1.2 and 2.2.

2°. Problem 1.1 (2.1) is solved by a strategy U^e extremal, for example, to set $W \cap Z(W)$, where Z is the bundle of trajectories starting from an initial position and corresponding to all possible admissible controls u and v. Problem 1.2 (2.2) too can be solved by a strategy V^e extremal, for example, to the bundle of all motions generated by strategy V_0 solving Problem 1.2 (2.2) (see the definition of set W).

3°. If Condition (2.1) is not fulfilled, then the assertion of Theorem 1.1 (2.1) remains valid, but Problem 1.1 (2.1) or Problem 1.2 (2.2) is now solved by a strategy "extremal" to a certain sequence of stable bridges inbedded one into the other /7.12/.

4°. If the bundles of trajectories of system (1.1)— (1.3) are precompact in $C = ([t_0, \vartheta]; \Phi)$, then from the assertion of Theorem 1.1 follows the alternative in game 1.1—1.2, where the distance to the set is measured in the metric induced by the metric in Φ. This fact can be proved analogously as in /6,7/. If

$$\sigma_2 = \sigma_2(x),\ a_{ij} = a_{ij}(x)\ a(t),\ 0 < \alpha \leqslant a(t) \leqslant \beta, \qquad f = f_1(t, x, y) + f_2(x, y)\ u(t) + f_3(x, y)\ v(t)$$

where functions f_1, f_2 and f_3 satisfy a Lipschitz condition in y, then under the fulfillment of certain regularity conditions /6,7,14,15/ the solutions of (1.1)— (1.3) are compact in

$$C([t_0, \vartheta]; \Phi)$$

5°. For certain classes of systems of form (1.1)— (1.3) analogous results are valid for the case of boundary controls

$$\partial y / \partial N + \sigma_2(x)\ y|_\Sigma = b(x)\ u(t) + c(x)\ v(t) + g(x)$$

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